Examples #8

Example 0.1. An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X = the number of points earned on the first part and Y = the number of points earned on the second part. Suppose that the joint pmf of X and Y is given in the accompanying table.

$p(x,y)$	$y=0$	$y=5$	$y=10$	$y=15$
$x=0$	.02	.06	.02	.10
$x=5$	.04	.15	.20	.10
$x=10$	.01	.15	.14	.01

1. If the score recorded in the grade book is the total number of points earned on the two parts, what is the expected recorded score $E(X+Y)$?
2. Compute the covariance for X and Y.
3. Compute ρ for X and Y.

Example 0.2. A gas station sells three grades of gasoline: regular, plus, and premium. Their prices are \$3.50, \$3.65, and \$3.80 per gallon, respectively. Let X_1, X_2, X_3 denote the number of gallons of each grade sold on a particular day. Suppose the X_i's are independent with

$$\mu_1 = 1000,\ \mu_2 = 500,\ \mu_3 = 300, \qquad \sigma_1 = 100,\ \sigma_2 = 80,\ \sigma_3 = 50.$$

1. Find the total revenue, Y, $E(Y)$, $V(Y)$, and $SD(Y)$.
2. If the X_i's are (approximately) normally distributed find the probability that revenue exceeds \$5000.
3. Would your calculations necessarily be correct if the X_i's were not independent? Explain.

Example 0.3. In a standby system, a component is used until it wears out and is then immediately replaced by another, not necessarily identical, component. (The second component is said to be "in standby mode," i.e., waiting to be used.) The overall lifetime of a standby system is just the sum of the lifetimes of its individual components. Let X and Y denote the lifetimes of the two components of a standby system, and suppose X and Y are independent exponentially distributed random variables with mean lifetimes 3 weeks and 4 weeks, respectively. Let W = X + Y, the system lifetime.

1. Find $E(W)$ and $V(W)$.
2. Is W exponentially distributed?
3. Find the pdf of W.
4. Find the probability that the system lasts more than its expected lifetime of 7 weeks.

Example 0.4 (Buffon's needle problem[1]). A plane is ruled with parallel lines a distance d apart. A needle of length l, with $l < d$, is tossed at random onto the plane. What is the probability that the needle intersects one of the parallel lines?

[1] *Georges Louis Buffon (1707–1784), the French naturalist who introduced and solved this famous problem. Beyond the needle problem, he is well known for his work on the distribution and expected remaining lifetimes of humans. These and many other studies were published in his 44-volume series, Histoire Naturelle (Natural History; 1749–1804).*