Examples #6

Example 0.1. Suppose the time spent by a randomly selected student at a campus computer laboratory has a gamma distribution with mean 20 minutes and variance $80\,\text{min}^2$.

(a) What are the values of the parameters α and β?

(b) What is the probability that a student uses the laboratory for at most 24 minutes?

Example 0.2. In studies of anticancer drugs it was found that if mice are injected with cancer cells, the survival time can be modeled with the exponential distribution. Without treatment the expected survival time was $\mu = 10$ h. What is the probability that

(a) a randomly selected mouse will survive at least 8 h? At most 12 h?

(b) the survival time of a mouse exceeds the mean value by more than 2 standard deviations?

Example 0.3. The lifetime X (in hundreds of hours) of a type of vacuum tube has a Weibull distribution with parameters $\alpha = 2$ and $\beta = 3$.

(a) Compute $E(X)$ and $V(X)$.

(b) Compute $P(1.5 \leq X \leq 6)$.

(c) What is the median lifetime of such tubes?

Example 0.4. A theoretical justification based on a material failure mechanism underlies the assumption that the ductile strength X of a material has a lognormal distribution. Suppose the parameters are $\mu = 5$ and $\sigma = 0.1$.

(a) If ten different samples of an alloy steel of this type were subjected to a strength test, how many would you expect to have strength of at least 125?

(b) If the smallest 5% of strength values were unacceptable, what would the minimum acceptable strength be?

Example 0.5. What condition on α and β is necessary for the standard beta probability density function to be symmetric?

Example 0.6. Let X have the pdf

$$f_X(x) = \frac{2}{x^3}, \quad x > 1.$$

Find the pdf of $Y = \sqrt{X}$.

Example 0.7. The variation in a certain electrical current source X (in milliamps) can be modeled by the pdf

$$f(x) = 1.25 - .25x, \quad 2 \leq x \leq 4.$$

If this current passes through a 220-ohm resistor, the resulting power Y (in microwatts) is given by the expression $Y = 220X^2$. Find the pdf of Y.

Example 0.8. Consider a standard normal rv Z with $Y = Z^2$. Find the pdf of Y.

Example 0.9. (a) If a measurement error X is uniformly distributed on $[-1, 1]$, find the pdf of $Y = |X|$, which is the magnitude of the measurement error.

(b) If $X \sim \text{Unif}[-1, 3]$, find the pdf of $Y = X^2$.