nurdan kar @ bilkent edu tr # 6. STATISTICS AND SAMPLING DISTRIBUTIONS # In this chapter, we will discuss concepts for transitions between probability and inferential statistics. 6.1. Statistics and their distributions Thus for we have used the rus (X) to define the theoretical probability distribution (the expected behavior) of the dota before it is collected Now, we use the observed values (x;) to colculate sample statistic ( like the sample mean or variance ) and perform inference after the data is available Sample observations change from one sample to another As a result any statistic used to estimate a population characteristic will almost never equal the true value. Definition. A statistic is any quantity whose value can be calculated from sample data. Prior to obtaining data, there is uncertainty as to what value of any particular statistic will result. Therefore, a statistic is a ru and will be denoted by an appercase letter a lowercase letter is used to represent the calculated or observed value of the statistic (e.g. sample mean X, sample stand devi. S) Because a statistic is a ru, it has its own probability dist this dist depends on not only the population dist, and the sample size in but the sampling method. Next definition is a commonly used sampling method. Definition The Eus X., X., X. are soid to form a climple random sample of size of if 1. The X; s are independent rus. 2. Every X; has the same prob. dist. also referred to as being independent identically distributed (iid)

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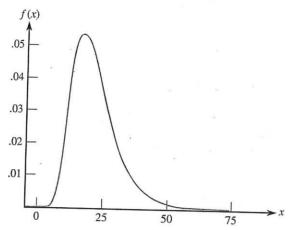
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**Example 6.4** Consider a simulation experiment in which the population distribution is quite skewed. Figure 6.5 shows the density curve for lifetimes of a certain type of electronic control. This is actually a lognormal distribution with  $E[\ln(X)] = 3$  and  $V[\ln(X)] = 0.16$ ; that is,  $\ln(X)$  is normal with mean 3 and standard deviation 0.4.



**Figure 6.5** Density curve for the simulation experiment of Example 6.4: a lognormal distribution with E(X) = 21.76 and V(X) = 82.14

Imagine the statistic of interest is the sample mean,  $\overline{X}$ . For any given sample size n, we repeat the following procedure k times:

- Generate values  $x_1, ..., x_n$  from a lognormal distribution with the specified parameter values; equivalently, generate  $y_1, ..., y_n$  from a N(3, 0.4) distribution and apply the transformation  $x = e^y$  to each value.
- Calculate and store the sample mean  $\bar{x}$  of the *n* x-values.

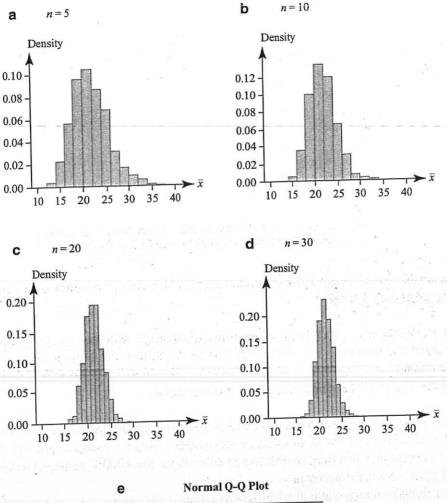
We performed this simulation experiment at four different sample sizes: n = 5, 10, 20, and 30. The experiment utilized k = 1000 replications (a very modest value) for each sample size. The resulting histograms along with a normal probability plot from R for the  $1000 \ \bar{x}$  values based on n = 30 are shown in Figure 6.6 on the next page.

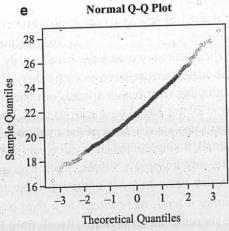
The first thing to notice about the histograms is that each one is centered approximately at the mean of the population being sampled,  $\mu_X = e^{3+0.16/2} \approx 21.76$ . Had the histograms been based on an unending sequence of  $\bar{x}$  values, their centers would have been exactly at the population mean.

Second, note the spread of the histograms relative to each other. The smaller the value of n, the greater the extent to which the sampling distribution spreads out about the mean value. This is why the histograms for n = 20 and n = 30 are based on narrower class intervals than those for the two smaller sample sizes. For the larger sample sizes, most of the  $\bar{x}$  values are quite close to  $\mu_X$ . This is the effect of averaging. When n is small, a single unusual x value can result in an  $\bar{x}$  value far from the center. With a larger sample size, any unusual x values, when averaged in with the other sample values, still tend to yield an  $\bar{x}$  value close to  $\mu_X$ . Combining these insights yields an intuitively-appealing result:  $\bar{X}$  based on a large n tends to be closer to  $\mu$  than does  $\bar{X}$  based on a small n.

Third and finally, consider the shapes of the histograms. Recall from Figure 6.5 that the population from which the samples were drawn is quite skewed. But as the sample size n increases, the distribution of  $\overline{X}$  appears to become progressively less skewed. In particular, when n = 30 the

distribution of the  $1000 \, \bar{x}$  values appears to be approximately normal, a fact validated by the normal probability plot in Figure 6.6e. We will discover in the next section that this is part of a much broader phenomenon known as the Central Limit Theorem: as the sample size n increases, the sampling distribution of  $\overline{X}$  becomes increasingly normal, irrespective of the population distribution from which values were sampled.





**Figure 6.6** Results of the simulation experiment of Example 6.4: (a)  $\overline{X}$  histogram for n = 5; (b)  $\overline{X}$  histogram for n = 10; (c)  $\overline{X}$  histogram for n = 20; (d)  $\overline{X}$  histogram for n = 30; (e) normal probability plot for n = 30 (from R)

5

Example. The amount of time that a potient spends in a certain outpatient Surgery is a rv with a mean value of 4.5 h and a standard deviation of 1.4h Let X1,..., X25 be the times for a random sample of 25 potients. a) Find  $E(T_0)$ ,  $E(\vec{X})$ ,  $G_{T_0}$ , and  $G_{\vec{X}}$ . b) If Xi's are normally distributed find the prob that the total time exceeds 120 h and the corresponding prob of overage time. Solution a)  $E(T_0) = n\mu = 25(45) = 112.56$ G = GVN = 14-25 = 7 E(X) = 45 h b) P(T,>120) = 1-P(T,6/20) 1- 5/120-112.5 1- 1 (1.07) overage time is 120 = 4.8 h  $P(|\bar{x} > 4.8) = 1 - P(\bar{x} \leq 4.8)$ 1- E 48 - 45 = 1- \(\varP(1.07)\) The Central Limit Theorem The central limit theorem shows that many natural phenomena obey approximately a standard normal distribution. In natural processes, the changes that take place are often the result of a sum is many insignificant random factors. Although the effect of each of these foctors, seperately, may be ignored the effect of their sum in general may not. Therefore, the study of distributions of sums

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normally distributed.  The following corollary is about the properties of $\beta$ .  Corollary. Consider an event $A$ with $p = P(A)$ . Let $X$ be the number of times $A$ occurs when the experiment is repeated a independent time.,  and define $\beta = P(A) = X$ 2. The possible outcomess $S$ and $F$ .  Then, $B = P(A) = X$ $C(S(A)) $	ropertion of succ	ssesip). By CLT, when a is large, both the	total
The following corollary is about the properties of $\beta$ .  Corollary Consider an event $A$ with $\rho=P(A)$ . Let $X$ be the number of times $A$ occurs when the experiment is repeated $\alpha$ independent time, and define $\beta=P(A)=X$ 2. Two possible outcomess $\beta$ and $\beta$ .  Then, $\beta=P(A)=X$ 2. Two possible outcomess $\beta$ and $\beta$ .  Then, $\gamma=P(A)=\gamma$ 1. E( $\gamma=P(A)=\gamma$ 2. Then, $\gamma=P(A)=\gamma$ 3. Pis const. trials $\gamma=P(A)=\gamma$ 3. As a increases, the distribution of $\gamma=P(A)=\gamma$ approaches a normal dist.		V not the morphe proportion P = X one app	roximately
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Corollary. Consider an event A with $\rho=P(A)$ . Let X be the number of times A occurs when the experiment is repeated a independent time, and define $\hat{\rho}=\hat{\rho}(A)=X \qquad \qquad 1. \ \text{Fixed numb. of trials in a possible outcomess s and F.}$ Then, $1. \ E(\hat{P})=\rho  \Rightarrow E(X(A))=X \qquad \qquad 2. \ \text{Then,}  X\times \text{Bin}(n,\rho).$ Independent trials $1. \ E(\hat{P})=\rho  \Rightarrow E(X(A))=(\alpha E(X)=V_0(n\rho)=\rho)  \text{Then,}  X\times \text{Bin}(n,\rho).$ $2. \ V(\hat{P})=P(I-P)  \text{and}  G_P=(P(I-P))$	normally distributed		
Corollary. Consider an event A with $\rho=P(A)$ . Let X be the number of times A occurs when the experiment is repeated a independent time, and define $\hat{\beta}=\hat{P}(A)=X$ In Fixed numb, of trials in 2. Two possible outcomess S and F.  Then, $1: E(\hat{P})=p \rightarrow E(X/n)=X = (E(X)=V_0(n\rho)=p \text{ Then, } X_N \text{ Bin}(n,p).$ 2. $V(\hat{P})=P(I-P)$ and $G_P=\int P(I-P)$	The following core	llary is about the properties of P.	
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2. $V(\hat{P}) = P(I-P)$ and $G_{\hat{P}} = P(I-P)$ 3. As a increases, the distribution of $\hat{P}$ approaches a normal dist.	Then,	4. Independent trial	\$
2. $V(\hat{P}) = P(I-P)$ and $G_{\hat{P}} = P(I-P)$ 3. As a increases, the distribution of $\hat{P}$ approaches a normal dist.	1 = (8) = 0	$\rightarrow E(X_0) = (0E(X) = V_0(00) = 0$ Then, $X \land Bin$	(n,12).
3. As a increases, the distribution of P approaches a normal dist.			
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Two Bernoulli distributions ( almost symmetric; ( very skewed) It Is close to . 5 The conditions np > 10 and n(1-p) > 10 more sure the sample size is big enough so we will get a normal dist when we consider only skewness in the underlying Bernoulli distribution. CLT can also be applied to: 8 Poisson, when u is large 18 Negative binomial, when r is large 1 Gamma, when as is large. Another application of CLT is stated below. Proposition. Let X, ..., X, be random sample from a distribution for which only positive values are possible [P(x,>0)=1]. Then if n is sufficiently large, the product  $H=X_1X_2\cdots X_n$  has approximately a lognormal distribution; that is lo (4) has a normal distribution The law of large numbers Law of large numbers. If X, X2, ..., Xn 75 a random sample from a distribution with mean u, then & converges to u 1. In mean square: E[(X-M)2] - a as a - 0 2. To probability: P(IX+MI>E) - a as n + 00 for any E>a. Proof. L. Recall the first proposition in this section. If X.,..., Xn Is a random sample from a distribution with mean u and standard deviotion C, then  $E(\bar{X}) = \mu$  and  $V(\bar{X}) = G^2/n$ . As a increases, the  $E(\bar{x})$  remains at  $\mu$  but the  $V(\bar{x})$  approaches zero; that is,  $E[(\bar{X}-\mu)]^2 = \chi(\bar{X}) = C^2 \rightarrow 0$  whe say that  $\bar{X}$  converges in mean square

to  $\mu$  because the mean of the squared difference between  $\overline{k}$  altivilloize and  $\mu$  goes to zero.

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6.3. The  $\chi^2$ , t, and F distributions this section, we present three distributions closely related to the normal: the chi-squared  $(\chi^2)$ , t, and E distributions The chi-squared distribution The chi-squared dist, is one of most useful dist. for hypothesis testing. Definition. For a positive integer is, let Z., ..., Z. be iid standard normal rus. Then the chi-squared distribution with a degrees of freedom (df) is defined to be the distribution of the ru  $X = Z^2 + \dots + Z^2$ This will sometimes be denoted by X ~ X, Novel, let us find the pdf of this distribution. Consider the co N=1. Then,  $X=Z_1$  and the colf of X $P(x) = P(X \le x) = P(Z^2 \le x) = P(-\sqrt{X} \le Z^2 \le \sqrt{X}) = \overline{P}(\sqrt{X}) - \overline{P}(-\sqrt{X})$ = 2 \(\overline{\pi}(\sqrt{\pi'}) - (\). By differentiation,  $f(x) = F'(x) = 2 \, \overline{p}'(\sqrt{x}) \, 1 + 0 = \phi(\sqrt{x}) \, 1$ 1X 12T 1X gamma pdf  $\leftarrow$  =  $f(x: \frac{1}{2}, 2)$ Here, we found 21 pdf. To generalize to any number of degrees of freedom, we use the maf of the gamma dist.  $M(t) = (1-\beta t)^{-1}$ Then, the mof of 2,7 when Z N N(0,1) Is M(+) = (1-2+) = By the def. of the chi-squared distribution and the properties of mgfs, for X × X we obtain  $M_{x}(t) = M_{z_{1}}(t) \cdot \dots \cdot M_{z_{1}}(t) + (1-2t)^{\frac{1}{2}} \cdot \dots \cdot (1-2t)^{\frac{1}{2}} = (1-2t)^{\frac{1}{2}}$ gamma mgf with

Proposition. The pof of the 12 distribution is Moreover, if  $x \sim 70^2$  then E(x) = 0, V(x) = 2v, and  $M_x(t) = (1-2t)^2$ In this proposition, E(K) and V(K) is obtained by u= a 3 = 2 . 2 = 19, c2 = x32 = x3 22 = 2x9 f(x, 9) ldf + pdf is unbounded r exponential with N=2. f(x)=1/2 e 1/2, x>0 If 1932, the dist is skewled, 2df but it becomes more symmetri as the de increases Proposition. 1. If  $X_3 = X_1 + X_2$ ,  $X_1$  and  $X_2$  are independent.  $X_1 \sim \chi_{q_1}^2$ , and  $X_2 \sim \chi_{q_2}^2$ , then  $X_3 \sim \chi_{q_1+q_2}^2$ . 2. If X3 = X, + X2, X, and X2 are independent, X, N X2 and  $X_3 \sim X_{u_3}^2$  with  $x_3 > v_1$ , then  $X_2 \sim X_{u_3 + v_4}^2$ Here, first statement says that chi-squared dist is additive. The second statement Joys that the distribution has the substractive property.

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