Examples #3

Example 0.1 (Binomial distribution)**.** Suppose a state has $500,000$ licensed drivers, of whom $400,000$ are insured. A sample of ten drivers is chosen without replacement. The ith trial is labeled S if the ith driver chosen is insured. Can the distribution of the number of insured drivers in the sample be modeled by a Binomial distribution?

Example 0.2 (Binomial pmf)**.** A restaurant serves 8 entrées of fish, 12 of beef, and 10 of poultry. If customers select from these entrées randomly, what is the probability that two of the next four customers order fish entrées?

Example 0.3 (Binomial cdf)**.** Suppose that 20% of all copies of a particular textbook fail a binding strength test. Let X denote the number among 15 randomly selected copies that fail the test. Then X has a binomial distribution with $n = 15$ and $p = 0.2$. Find

(a) the probability that at most 8 fail the test;

(b) the probability that between 4 and 7, inclusive, fail the test.

Example 0.4 (Binomial mean)**.** Two proofreaders, Ruby and Myra, read a book independently and found r and m misprints, respectively. Suppose that the probability that a misprint is noticed by Ruby is p and the probability that it is noticed by Myra is q, where these two probabilities are independent. If the number of misprints noticed by both Ruby and Myra is b, estimate the number of unnoticed misprints.[1]

[1] This problem was posed and solved by George Pólya (1888–1985) in the January 1976 issue of the *American Mathematical Monthly.*

Example 0.5 (Poisson distribution)**.** Every week the average number of wrong-number phone calls received by a certain mail-order house is seven. What is the probability that they will receive

(a) two wrong calls tomorrow;

(b) at least one wrong call tomorrow?

Example 0.6 (Poisson distribution)**.** The atoms of a radioactive element are randomly disintegrating. If every gram of this element, on average, emits 3.9 alpha particles per second, what is the probability that during the next second the number of alpha particles emitted from 1 gram is

(a) at most 6;

(b) at least 2;

(c) at least 3 and at most 6?

Example 0.7 (Poisson process)**.** Let $N(t)$ be the number of earthquakes that occur at or prior to time t worldwide. Suppose that $\{N(t) : t \geq 0\}$ is a Poisson process with rate λ, and that the probability that the magnitude of an earthquake on the Richter scale is 5 or more is p. Find the probability of k earthquakes of such magnitudes at or prior to t worldwide.